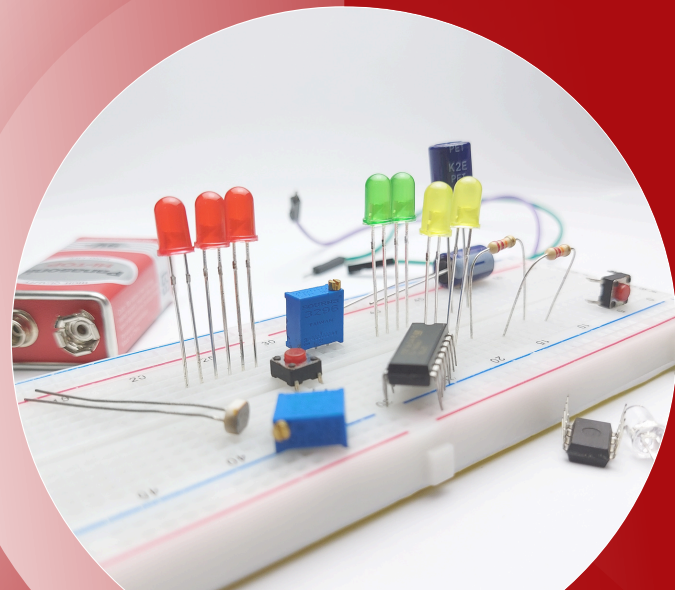
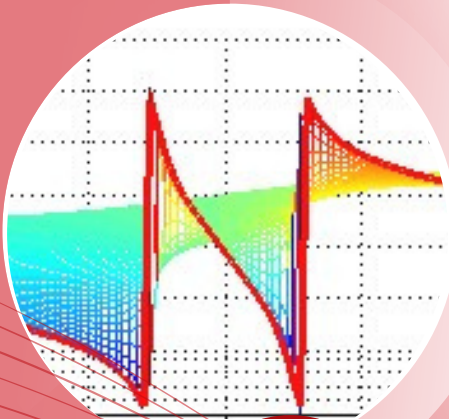


# Laplace Transform

## Notes & Exercises Series



$$\mathcal{L}\{f\}(s) = \int_0^{\infty} f(t)e^{-st} dt$$

# Laplace Transform

**Notes & Exercises Series**

**AUTHORS:**

Maziah binti Omar

Masniza binti Musa

Nang Saruni binti Nek Ali

# Preface

This e-book edition is designed to help teachers guide students through notes and worked examples, while also providing simple practice worksheets to enhance their skills and understanding of the Laplace Transform topic. The author wishes to express gratitude to all educators and learners who make use of this resource. It is hoped that this book will serve as a valuable aid in the teaching and learning process, and inspire students to continue exploring the beauty of mathematics with confidence and curiosity.



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# Laplace Transform

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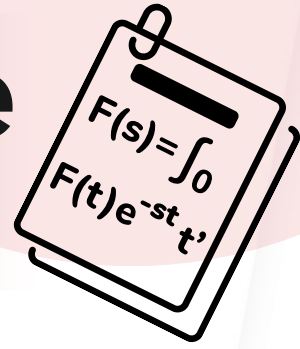


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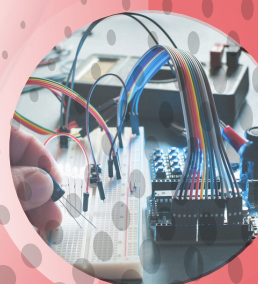
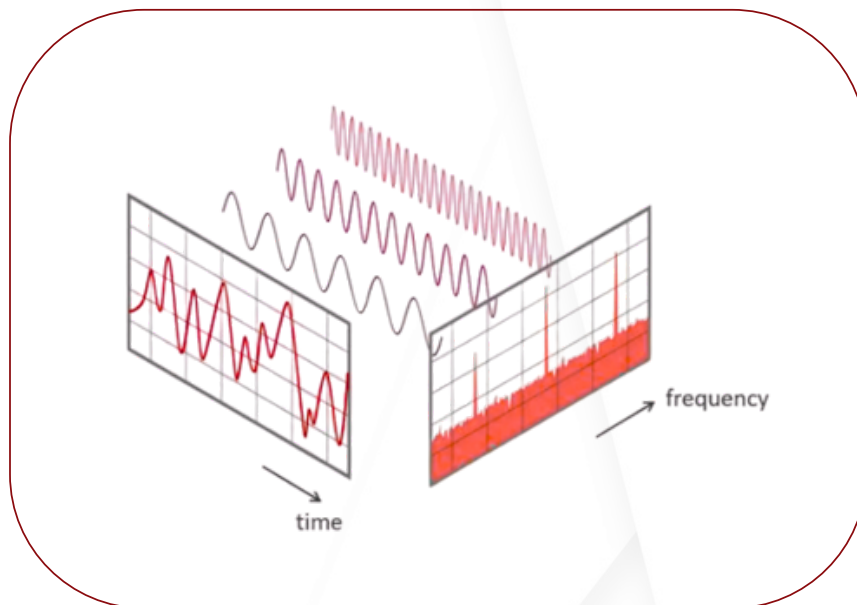


# What is the Laplace Transform?

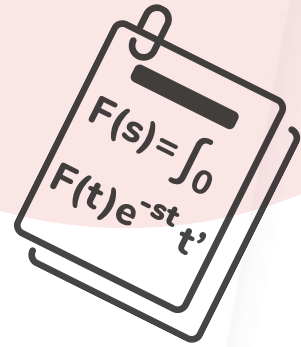


Laplace Transform is a mathematical technique that converts a function from the time domain into the frequency domain. In the time domain, functions are usually expressed in terms of  $t$  (time), while in the frequency domain they are represented using the variable  $s$ . By applying this transformation, problems that involve differentiation and integration can often be simplified into algebraic equations, which are much easier to solve.

$$F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$



# Definition of Laplace Transforms


$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

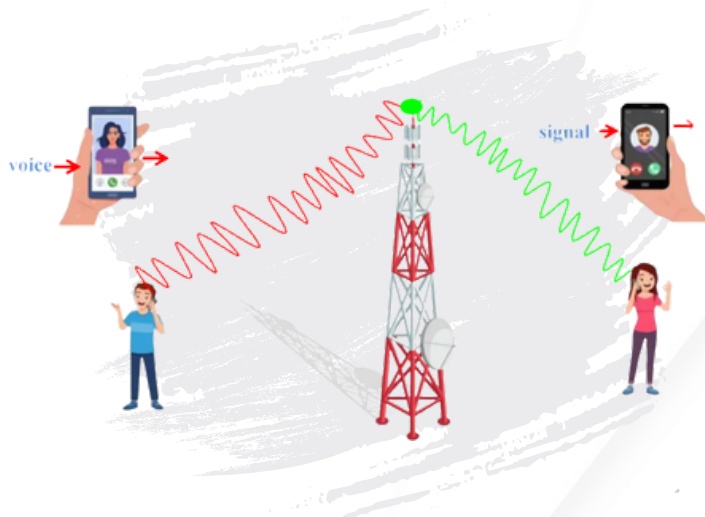
The Laplace Transform of a function is defined as:

$$F(s) = L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

where:

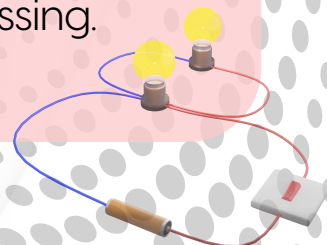
- $f(t)$  is the original function (in time domain)
- $F(s)$  is the transformed function (in frequency domain)

Solving a Laplace Transform in the form of using the definition of Laplace Transform, we need to solve the equation .



## EXTRA INFO!

Engineers and scientists often use Laplace Transform in fields such as electrical circuits, control systems, vibrations, and signal processing.



# Definition of Laplace Transform

## Example 1

By using the definition  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ , find Laplace Transform for  $f(t) = 3$ .

**SOLUTION:**

$$f(t) = 3 \text{ (constant)}$$

$$\begin{aligned} F(s) &= \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} (3) dt \\ &= 3 \int_0^{\infty} e^{-st} dt \\ &= \left[ \frac{3e^{-st}}{-s} \right]_0^{\infty} \\ &= \frac{3e^{-\infty}}{-s} - \frac{3e^0}{-s} \\ &= 0 - \frac{3}{-s} \\ &= \frac{3}{s} \end{aligned}$$

Bring 3 to the front,  
then integrate the  $e^{-st}$ .

## Example 2

By using the definition  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ , find Laplace Transform for  $f(t) = \frac{5}{2}$ .

**SOLUTION:**

$$f(t) = \frac{5}{2} \text{ (constant)}$$

$$\begin{aligned} F(s) &= \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} \left( \frac{5}{2} \right) dt \\ &= \frac{5}{2} \int_0^{\infty} e^{-st} dt \\ &= \left[ \frac{5e^{-st}}{-s(2)} \right]_0^{\infty} \\ &= \frac{5e^{-\infty}}{-2s} - \frac{5e^0}{-2s} \\ &= 0 - \frac{5}{-2s} \\ &= \frac{5}{2s} \end{aligned}$$

Bring  $\frac{5}{2}$  to the front,  
then integrate the  $e^{-st}$ .

# Definition of Laplace Transform

## Example 3

By using the definition  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ , find Laplace Transform for  $f(t) = e^{2t}$ .

**SOLUTION:**

$$\begin{aligned} F(s) &= \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} (e^{2t}) dt \\ &= \int_0^{\infty} e^{-st+2t} dt \\ &= \int_0^{\infty} e^{-(s-2)t} dt \\ &= \left[ \frac{e^{-(s-2)t}}{-(s-2)} \right]_0^{\infty} \\ &= \frac{e^{-(s-2)\infty}}{-(s-2)} - \frac{e^{-(s-2)0}}{-(s-2)} \\ &= \frac{e^{-\infty}}{-(s-2)} - \frac{e^0}{-(s-2)} \\ &= \frac{1}{-(s-2)} - \frac{1}{-(s-2)} \\ &= 0 - \frac{1}{-(s-2)} \\ &= \frac{1}{(s-2)} \end{aligned}$$

Factorize the power:  
 $-st + 2t = -(s-2)t$

# Definition of Laplace Transform

## Example 4

By using the definition  $F(s) = \int_0^{\infty} e^{-st} f(t) dt$ , find Laplace Transform for  $f(t) = 2t$ .

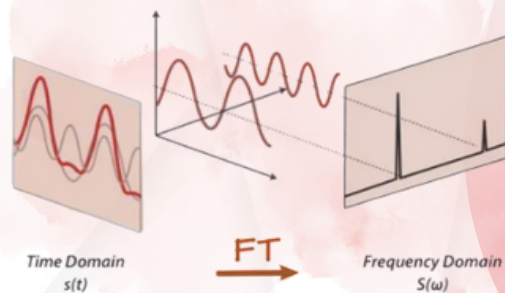
**SOLUTION:**

$f(t) = t$  (algebraic function)

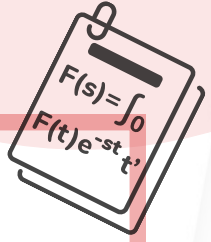
$$\begin{aligned} F(s) &= \int_0^{\infty} e^{-st} f(t) dt \\ &= \int_0^{\infty} e^{-st} (2t) dt \\ &= uv - \int_0^{\infty} v du \\ &= (2t) \left( \frac{e^{-st}}{-s} \right) - \int_0^{\infty} \frac{e^{-st}}{-s} 2 dt \\ &= \left[ -\frac{2te^{-st}}{s} + \frac{2e^{-st}}{-s^2} \right]_0^{\infty} \\ &= \left[ -\frac{2(\infty)e^{-\infty}}{s} - \frac{2e^{-\infty}}{s^2} \right] - \left[ -\frac{2(0)e^0}{s} - \frac{2e^0}{s^2} \right] \\ &= [0] - \left[ -0 - \frac{2}{s^2} \right] \\ &= \frac{2}{s^2} \end{aligned}$$

Use integration by parts to integrate:

$$\begin{aligned} u &= 2t & dv &= e^{-st} \\ \frac{du}{dt} &= 2, du = 2dt & \int dv &= \int e^{-st} \\ du &= 2dt & v &= \frac{e^{-st}}{-s} \end{aligned}$$



## LET'S PRACTISE!



1. Find the Laplace Transform for the following functions using the definition of Laplace Transform.

a)  $f(t) = p$

**Answer:**  $F(s) = \frac{p}{s}$

b)  $f(t) = 5n$

**Answer:**  $F(s) = \frac{5n}{s}$

c)  $f(t) = \frac{3}{5}$

**Answer:**  $F(s) = \frac{3}{5s}$

d)  $f(t) = e^{6t}$

**Answer:**  $F(s) = \frac{1}{s-6}$

e)  $f(t) = \frac{e^{3t}}{2}$

**Answer:**  $F(s) = \frac{1}{2(s-3)}$

f)  $f(t) = 4e^{-5t}$

**Answer:**  $F(s) = \frac{4}{s+5}$

g)  $f(t) = \frac{t}{7}$

**Answer:**  $F(s) = \frac{1}{7s^2}$

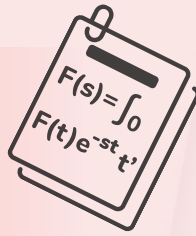
## POP QUIZ

To find out your level of understanding, let's take a



POP QUIZ DEFINITION OF  
LAPLACE TRANSFORM

# Laplace Transforms using table



To find the Laplace Transform, we can also use the table of Laplace Transform as follows:

LAPLACE TRANSFORM					
No.	$f(t)$	$F(s)$	No.	$f(t)$	$F(s)$
1.	$a$	$\frac{a}{s}$	13.	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
2.	$at$	$\frac{a}{s^2}$	14.	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
3.	$t^n$	$\frac{n!}{s^{n+1}}$	15.	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
4.	$e^{at}$	$\frac{1}{s-a}$	16.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
5.	$e^{-at}$	$\frac{1}{s+a}$	17.	$e^{at} \sinh \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
6.	$te^{-at}$	$\frac{1}{(s+a)^2}$	18.	$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2 - \omega^2}$
7.	$t^n \cdot e^{at}, n = 1, 2, 3 \dots$	$\frac{n!}{(s-a)^{n+1}}$	19.	$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2 - \omega^2}$
8.	$t^n \cdot f(t)$	$(-1)^n \frac{d^n}{ds^n} [F(s)]$	20.	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
9.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	21.	$\int_0^t f(u) du$	$\frac{F(s)}{s}$
10.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	22.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
11.	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	23.	First derivative $\frac{dy}{dt}, y'(t)$	$sY(s) - y(0)$
12.	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	24.	Second derivative $\frac{d^2y}{dt^2}, y''(t)$	$s^2Y(s) - sy(0) - y'(0)$



Video How to use Laplace Transform table

Video

# Laplace Transform using table

## Example 5

Find the Laplace Transform of the following functions by using the table of Laplace Transform :

a)  $f(t) = \frac{5}{2}$

c)  $f(t) = \sin 3t$

b)  $f(t) = e^{4t}$

d)  $f(t) = 2t^2$

**SOLUTION:**

$$\begin{aligned} \text{a) } f(t) &= \frac{5}{2} \\ F(s) &= L\left\{\frac{5}{2}\right\} \\ &= \frac{5}{2s} \end{aligned}$$

Using table no. 1 :

$$f(t) = a, \text{ so } F(s) = \frac{a}{s}$$

$$\begin{aligned} \text{b) } f(t) &= e^{4t} \\ F(s) &= L\{e^{4t}\} \\ &= \frac{1}{s-4} \end{aligned}$$

Using table no. 4 :

$$f(t) = e^{at}, \text{ so } F(s) = \frac{1}{s-a}$$

$$\begin{aligned} \text{c) } f(t) &= \sin 3t \\ F(s) &= L\{\sin 3t\} \\ &= \frac{3}{s^2-3^2} \\ &= \frac{3}{s^2-9} \end{aligned}$$

Using table no. 9 :

$$f(t) = \sin \omega t, \text{ so } F(s) = \frac{\omega}{s^2+\omega^2}$$

$$\begin{aligned} \text{d) } f(t) &= 2t^2 \\ F(s) &= L\{2t^2\} \\ &= 2\left(\frac{2!}{s^{2+1}}\right) \\ &= \frac{4}{s^3} \end{aligned}$$

Using table no. 3 :

$$f(t) = t^n, \text{ so } F(s) = \frac{n!}{s^{n+1}}$$

## LET'S PRACTISE!

Find the Laplace Transform for the following functions using the table of Laplace Transform.

a)  $f(t) = p$

Answer:  $F(s) = \frac{p}{s}$

b)  $f(t) = 5n$

Answer:  $F(s) = \frac{5n}{s}$

c)  $f(t) = \frac{3}{5}$

Answer:  $F(s) = \frac{3}{5s}$

d)  $f(t) = e^{-6t}$

Answer:  $F(s) = \frac{1}{s+6}$

e)  $f(t) = \frac{e^{3t}}{2}$

Answer:  $F(s) = \frac{1}{2(s-3)}$

f)  $f(t) = 4e^{-5t}$

Answer:  $F(s) = \frac{4}{s+5}$

g)  $f(t) = \frac{t}{7}$

Answer:  $F(s) = \frac{1}{7s^2}$

h)  $f(t) = \cos 8t$

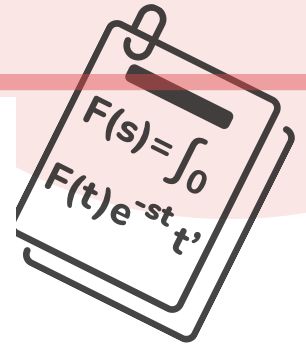
Answer:  $F(s) = \frac{s}{s^2+64}$

i)  $f(t) = 3\sin 2t$

Answer:  $F(s) = \frac{6}{s^2+36}$

j)  $f(t) = 3t^3$

Answer:  $F(s) = \frac{18}{s^4}$



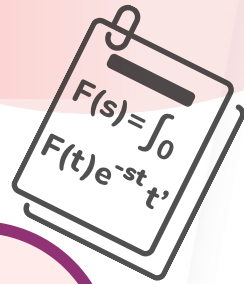
## POP QUIZ

To find out your level of understanding, let's take a pop quiz



POP QUIZ TABLE OF  
LAPLACE TRANSFORM

# Applying Laplace Transforms Properties



1

## THEOREM 1

Linearity of Laplace Transforms

2

## THEOREM 2

First Shifting

3

## THEOREM 3

Multiplication by  $t^n$

## EXTRA INFO!

### THEOREM 1

Allowing constants to be factored out and sums to be transformed individually

### THEOREM 2

Question must have:

$$L\{e^{at}f(t)\} \text{ or } L\{e^{-at}f(t)\}$$

### THEOREM 3

Question must have:

$$L\{f(t)t^n\} \text{ or } L\{t^n f(t)\}$$

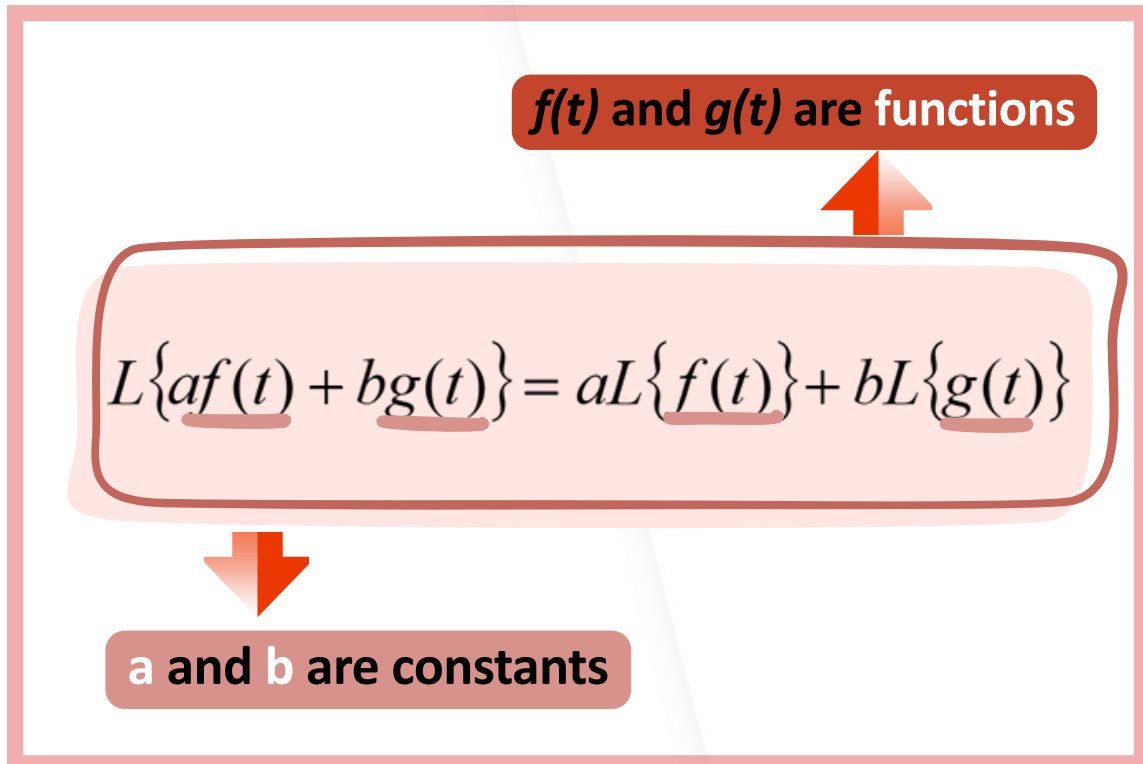


## THEOREM 1

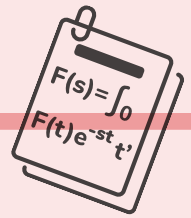
# Linearity of Laplace Transforms



Linearity of Laplace Transforms is a property that states the transform of a linear combination of functions is equal to the linear combination of their individual transforms, present as:



This property can be easily extended to more than two functions as shown from the above proof. With the linearity property, Laplace transform can also be called the linear operator.



## EXAMPLE 6

Find the Laplace transforms of the following functions by using the table of Laplace transform below:

- a)  $f(t) = \cos 3t - e^{-2t}$   
 b)  $f(t) = 6 + 2t^5 - \sinh 7t$

### SOLUTION:

Look at each term separately and try to find a Laplace transform by using the table of Laplace Transform for  $\cos 3t$  and  $e^{-2t}$

$$\begin{aligned} \text{a) } f(t) &= \cos 3t - e^{-2t} \\ F(s) &= L\{\cos 3t\} - L\{e^{-2t}\} \\ F(s) &= \frac{s}{s^2+3^2} - \frac{1}{s+2} \\ &= \frac{s}{s^2+9} - \frac{1}{s+2} \end{aligned}$$

$e^{-2t}$	Refer F5: $\frac{1}{s+a}$
$\cos 3t$	Refer F10: $\frac{s}{s^2+\omega^2}$

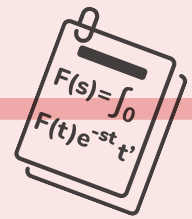
### SOLUTION:

$$\begin{aligned} \text{b) } f(t) &= 6 + 2t^5 - \sinh 7t \\ F(s) &= L\{6\} + 2L\{t^5\} - L\{\sinh 7t\} \\ F(s) &= \frac{6}{s} + 2\left\{\frac{5!}{s^{5+1}}\right\} - \frac{7}{s^2-7^2} \\ &= \frac{6}{s} + \frac{240}{s^6} - \frac{7}{s^2-49} \end{aligned}$$

- Look at each term separately and try to find a Laplace transform by using the table of Laplace Transform for  $6$ ,  $2t^5$  and  $\sinh 7t$

- Allowing constant to be factored out for  $2t^5$ .

$6$	Refer F1: $\frac{a}{s}$
$2t^5$	Refer F3: $\frac{n!}{s^{n+1}}$
$\sinh 7t$	Refer F15: $\frac{\omega}{s^2-\omega^2}$



## EXAMPLE 7

Apply the Linearity Theorem to find the Laplace Transforms of the following functions:

- $f(t) = 3 \sin \sqrt{2}t - e^{3t} \cosh 8t$
- $f(t) = \frac{2}{e^{6t}} + t^2 e^{4t} - 8t$
- $f(t) = t(12t - 2t^3)$

### SOLUTION:

$$\begin{aligned} \text{i. } f(t) &= 3 \sin \sqrt{2}t - e^{3t} \cosh 8t \\ F(s) &= 3L\{\sin \sqrt{2}t\} - L\{e^{3t} \cosh 8t\} \\ F(s) &= 3 \left\{ \frac{\sqrt{2}}{s^2 + (\sqrt{2})^2} \right\} - \frac{s-3}{(s-3)^2 - 8^2} \\ &= \frac{3\sqrt{2}}{s^2+2} - \frac{s-3}{(s-3)^2-64} \end{aligned}$$

- Look at each term separately and try to find a Laplace transform by using the table of Laplace Transform for  $\sin \sqrt{2}t$  and  $e^{3t} \cosh 8t$

- Allowing constant to be factored out for  $3 \sin \sqrt{2}t$

$3 \sin \sqrt{2}t$  Refer F9:  $\frac{\omega}{s^2 + \omega^2}$

$e^{3t} \cosh 8t$  Refer F19:  $\frac{s-a}{(s-a)^2 - \omega^2}$   
convert to  $s-a$  since  $e^{3t} : (a+)$

$$\begin{aligned} \text{ii. } f(t) &= \frac{2}{e^{6t}} + t^2 e^{4t} - 8t \\ f(t) &= 2e^{-6t} + t^2 e^{4t} - 8t \\ F(s) &= 2L\{e^{-6t}\} + L\{t^2 e^{4t}\} - L\{8t\} \\ F(s) &= 2 \left\{ \frac{1}{s+6} \right\} + \frac{2!}{(s-4)^{2+1}} - \frac{8}{s^2} \\ &= \frac{2}{s+6} + \frac{2}{(s-4)^3} - \frac{8}{s^2} \end{aligned}$$

Before solving, we must convert

$f(t) = \frac{2}{e^{6t}}$  into  $2e^{-6t}$

- Look at each term separately and try to find a Laplace transform by using the table of Laplace Transform for  $2e^{-6t}$ ,  $t^2 e^{4t}$  and  $8t$

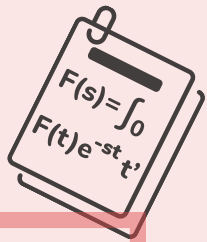
- Allowing constant to be factored out for  $2e^{-6t}$

$2e^{-6t}$  Refer F5:  $\frac{1}{s+a}$

$t^2 e^{4t}$  Refer F7:  $\frac{n!}{(s-a)^{n+1}}$

$8t$  Refer F2:  $\frac{a}{s^2}$

# Linearity of Laplace Transform



## CONTINUE...

$$\begin{aligned} \text{iii. } f(t) &= t(12t - 2t^3) \\ f(t) &= 12t^2 - 2t^4 \\ F(s) &= 12L\{t^2\} - 2L\{t^4\} \\ F(s) &= 12 \left\{ \frac{2!}{s^{2+1}} \right\} - 2 \left\{ \frac{4!}{s^{4+1}} \right\} \\ &= \underline{\underline{\frac{24}{s^3} - \frac{48}{s^5}}} \end{aligned}$$

Before solving, we need to expand the equation

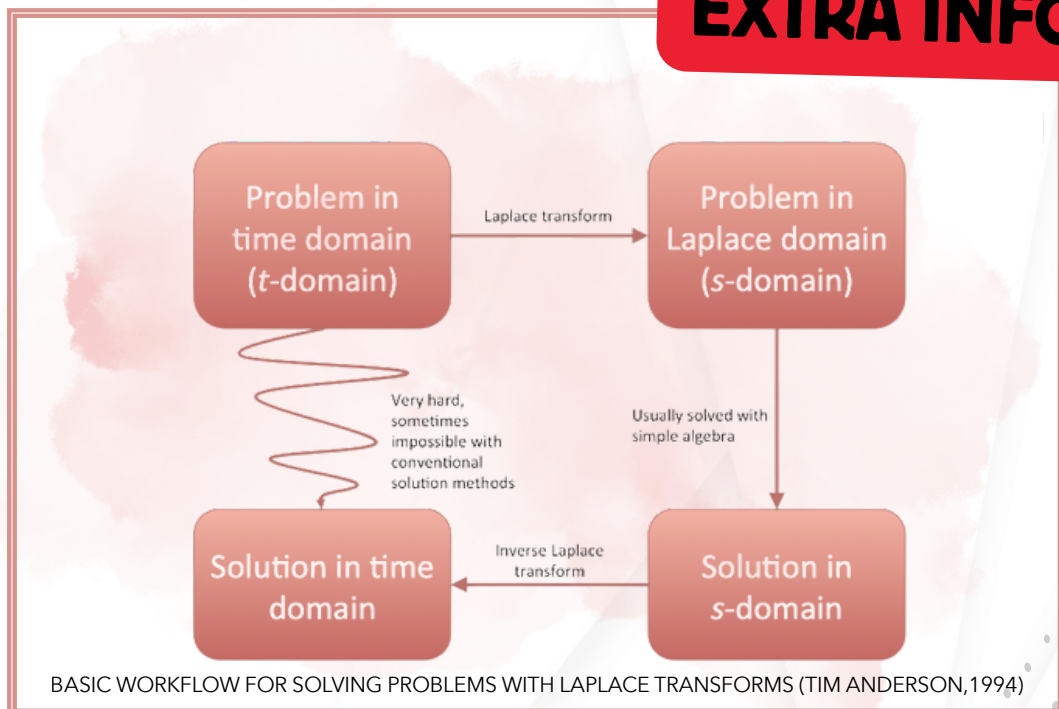
- Look at each term separately and try to find a Laplace transform by using the table of Laplace Transform for  $12t^2$  and  $2t^4$

- Allowing constants to be factored out for  $12t^2$  and  $2t^4$

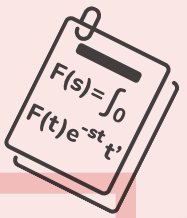
$$\begin{matrix} 12t^2 \\ 2t^4 \end{matrix}$$

Refer F3:  $\frac{n!}{s^{n+1}}$

## EXTRA INFO!



## LET'S PRACTISE!



1. Find the Laplace Transform for the following **linearity functions** using the table of Laplace Transforms.

a)  $f(t) = -9 + e^{-3t} + t^6$

**Answer:**  $\frac{-9}{s} + \frac{1}{s+3} + \frac{720}{s^7}$

b)  $f(t) = 2 \sin 6t - 3 \cos 5t$

**Answer:**  $\frac{12}{s^2+36} + \frac{3s}{s^2+25}$

c)  $f(t) = 7t + \cosh 3t - 5e^{2t}$

**Answer:**  $\frac{7}{s^2} + \frac{s}{s^2-9} - \frac{5}{s-2}$

d)  $f(t) = e^{-t} \cos 8t - t \sin 3t$

**Answer:**  $\frac{s+1}{(s+1)^2-64} - \frac{6s}{(s^2+9)^2}$

2. Solve the Laplace Transform for the following functions using the table of Laplace Transforms.

i.  $f(t) = 5te^{2t} + 5t^3 + \frac{1}{e^{9t}}$

**Answer:**  $\frac{5}{(s-2)^2} + \frac{30}{s^4} + \frac{1}{s+9}$

ii.  $f(t) = \cos 3t + t \cos 7t$

**Answer:**  $\frac{s}{s^2+9} + \frac{s^2-49}{(s^2+49)^2}$

iii.  $f(t) = 3t(1 + t + t^2)$

**Answer:**  $\frac{3}{s^2} + \frac{6}{s^3} + \frac{18}{s^4}$

iv.  $f(t) = \frac{e^{7t}}{6} + 10 - 5e^{4t} \sinh \sqrt{3} t$

**Answer:**  $\frac{1}{6(s+7)} + \frac{10}{s} - \frac{5\sqrt{3}}{(s-4)^2-3}$

## POP QUIZ

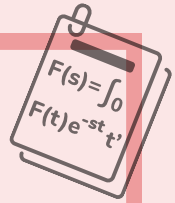
To find out your level of understanding, let's take a pop quiz



POP QUIZ LINEARITY

## THEOREM 2

# First Shifting



We know that  $f(t)$  is a function and  $L\{f(t)\} = F(s)$   
 $a$  is a constant. The first shifting theorem states that:

$$\begin{aligned} L\{e^{at}f(t)\} &= F(s - a) \quad \text{where } s \rightarrow (s - a) \\ L\{e^{-at}f(t)\} &= F(s + a) \quad \text{where } s \rightarrow (s + a) \end{aligned}$$

The notation  $F(s - a)$  and  $F(s + a)$  can be found when we replace  $(s - a)$  or  $(s + a)$  in the  $F(s)$  function.

## HOW TO SOLVE FIRST SHIFTING

### STEP 1

Determine  $f(t)$  and  $a$ , where :

$f(t)$  = all term except  $e^{at}$  or  $e^{-at}$

$a$  = refer to power of  $e^{at}$  or  $e^{-at}$

$$L\{e^{at}f(t)\}$$

$$L\{e^{-at}f(t)\}$$

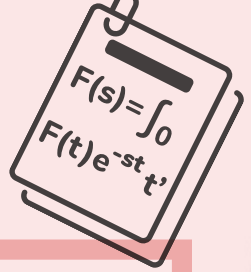
### STEP 2

Find  $f(t)$  by using Table of Laplace Transform

### STEP 3

Replace  $s$  with  $(s - a)$  or  $(s + a)$  based on the answer in **STEP 2**

# First Shifting



## EXAMPLE 8

Find the following Laplace transform by using the first shifting theorem:

- a)  $L\{-2e^{5t}\}$
- b)  $L\{t^3e^{-7t}\}$
- c)  $L\{2e^{3t} \sin 6t\}$

### SOLUTION:

a)  $L\{-2e^{5t}\}$

**STEP 1:**  $f(t) = -2$ ,  $a = 5t$  (refer to  $e^{5t}$ )

**STEP 2:** Find  $f(t) = -2$  by using Table of Laplace Transform

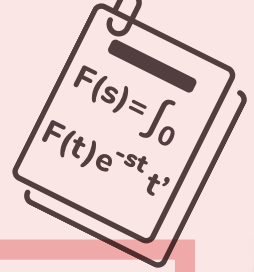
$$L\{-2\} = \frac{-2}{s}$$

-2	Refer F1: $\frac{a}{s}$
----	-------------------------

**STEP 3:** Replace  $s$  with  $s - 5$  based on the answer in **STEP 2**  $\rightarrow a = 5t$

$$L\{-2e^{5t}\} = \frac{-2}{s - 5}$$

# First Shifting



## CONTINUE...

b)  $L\{t^3 e^{-7t}\}$

**STEP 1:**  $f(t) = t^3$ ,  $a = -7t$  (refer to  $e^{-7t}$ )

**STEP 2:** Find  $f(t) = t^3$  by using Table of Laplace Transform

$$\begin{aligned}L\{t^3\} &= \frac{3!}{s^{3+1}} \\ &= \frac{6}{s^4}\end{aligned}$$

$t^3$  Refer F3:  $\frac{n!}{s^{n+1}}$

**STEP 3:** Replace  $s$  with  $s + 7$  based on the answer in **STEP 2**  $\rightarrow a = -7t$

$$L\{t^3 e^{-7t}\} = \frac{6}{(s+7)^4}$$

c)  $L\{2e^{3t} \sin 6t\}$

**STEP 1:**  $f(t) = 2\sin 6t$ ,  $a = 3t$  (refer to  $e^{3t}$ )

**STEP 2:** Find  $f(t) = 2\sin 6t$  by using Table of Laplace Transform

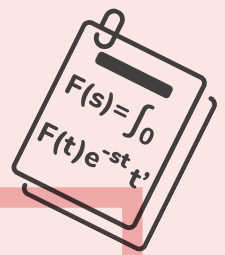
$$\begin{aligned}L\{2\sin 6t\} &= 2 \left\{ \frac{6}{s^2 + 6^2} \right\} \\ &= \frac{12}{s^2 + 36}\end{aligned}$$

$2\sin 6t$  Refer F9:  $\frac{\omega}{s^2 + \omega^2}$

**STEP 3:** Replace  $s$  with  $s - 3$  based on the answer in **STEP 2**  $\rightarrow a = 3t$

$$L\{2e^{3t} \sin 6t\} = \frac{12}{(s-3)^2 + 36}$$

# First Shifting



## EXAMPLE 9

By using the **first shifting theorem**, find the following Laplace transform:

- a)  $L\{3t^4 e^{\frac{t}{2}}\}$
- b)  $L\{e^{-3t} \cosh 8t\}$

### SOLUTION:

a)  $L\{3t^4 e^{\frac{t}{2}}\}$

**STEP 1:**  $f(t) = 3t^4$ ,  $a = \frac{t}{2} \rightarrow \frac{1}{2}t$  (refer to  $e^{\frac{t}{2}}$ )

**STEP 2:** Find  $f(t) = 3t^4$  by using Table of Laplace Transform

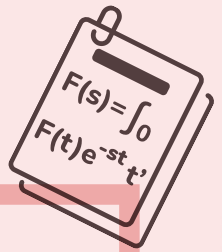
$$\begin{aligned} L\{3t^4\} &= 3 \left\{ \frac{4!}{s^{4+1}} \right\} \\ &= \frac{72}{s^5} \end{aligned}$$

$3t^4$  Refer F3:  $\frac{n!}{s^{n+1}}$

**STEP 3:** Replace  $s$  with  $s - \frac{1}{2}$  based on the answer in **STEP 2**  $\rightarrow a = \frac{1}{2}t$

$$L\left\{3t^4 e^{\frac{t}{2}}\right\} = \frac{72}{\left(s - \frac{1}{2}\right)^5}$$

# First Shifting



**CONTINUE...**

b)  $L\{e^{-3t} \cosh 8t\}$

**STEP 1:**  $f(t) = \cosh 8t$  ,  $a = -3t$  (refer to  $e^{-3t}$ )

**STEP 2:** Find  $f(t) = \cosh 8t$  by using Table of Laplace Transform

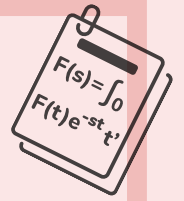
$$\begin{aligned} L\{\cosh 8t\} &= \frac{s}{s^2 - 8^2} \\ &= \frac{s}{s^2 - 64} \end{aligned}$$

$\cosh 8t$  Refer F16:  $\frac{s}{s^2 - \omega^2}$

**STEP 3:** Replace  $s$  with  $s + 3$  based on the answer in **STEP 2**  $\rightarrow a = -3t$

$$L\{e^{-3t} \cosh 8t\} = \frac{s + 3}{(s + 3)^2 - 64}$$

## LET'S PRACTISE!



Find the following Laplace transforms using the **first shifting theorem**:

a.  $L\{32e^{-t}\}$

**Answer:**  $\frac{32}{s+1}$

b.  $L\{e^{-9t} \sinh 5t\}$

**Answer:**  $\frac{5}{(s+9)^2 - 25}$

c.  $L\{\frac{2}{5}t^6 e^{-5t}\}$

**Answer:**  $\frac{288}{(s+5)^7}$

d.  $L\{6e^{2t} \cos \sqrt{3}t\}$

**Answer:**  $\frac{6(s-2)}{(s-2)^2 - 3}$

e.  $L\{2e^{6t}t^2\}$

**Answer:**  $\frac{4}{(s-6)^3}$

f.  $\cosh 9t e^{-7t}$

**Answer:**  $\frac{s+7}{(s+7)^2 - 81}$

## POP QUIZ

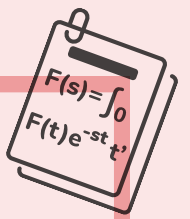
To find out your level of understanding, let's take a pop quiz



POP QUIZ First Shifting Theorem

### THEOREM 3

## Multiplication by $t^n$



The Multiplication by  $t^n$  states that the Laplace Transform of  $t^n$  multiplied by a function  $f(t)$  is equal to the negative derivative of the Laplace Transform of  $f(t)$  with respect to  $s$ .

Mathematically, if  $L\{f(t)\} = F(s)$ , then

$$L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n} F(s) \quad \text{where } n = 1, 2, 3, \dots$$

## How to Solve Multiplication by $t^n$

### STEP 1

Determine  $f(t)$  and  $n$ , where :

$f(t)$  = all term except  $t$

$n$  = refer to power of  $t$

$$L\{f(t)t^n\}$$

$$L\{t^n f(t)\}$$

### STEP 2

Find  $f(t)$  by using Table of Laplace Transform

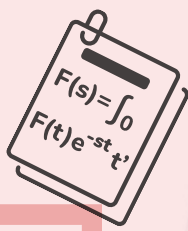
### STEP 3

Solve the equation by using Table of Laplace Transform (F8)

use :  $n$  from **STEP 1**

$F(s)$  from **STEP 2**

# Multiplication by $t^n$



## EXAMPLE 10

Find the following Laplace transform by using the **multiplication by  $t^n$  theorem**:

- a)  $L\{10t\}$
- b)  $L\{3te^{-3t}\}$
- c)  $L\{t^2e^{5t}\}$

### SOLUTION:

a)  $L\{10t\}$

**STEP 1:**  $f(t) = 10$ ,  $n = 1$  (refer to  $t$ )

**STEP 2:** Find  $f(t) = 10$  by using Table of Laplace Transform

$$L\{10\} = \frac{10}{s}$$
$$\therefore F(s) = \frac{10}{s}$$

10

Refer F1:  $\frac{a}{s}$

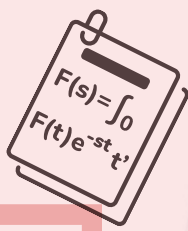
**STEP 3:** Solve the equation by using **F8**:  $n$  from **STEP 1**,  $F(s)$  from **STEP 2**

**F8**  $(-1)^n \frac{d^n}{ds^n} [F(s)]$

$$\begin{aligned} L\{10t\} &= (-1)^1 \frac{d^1}{ds^1} \left[ \frac{10}{s} \right] \\ &= (-1) \frac{d}{ds} [10s^{-1}] \\ &= (-1) (-1) 10s^{-2} \\ &= 10s^{-2} \\ &= \frac{10}{s^2} \end{aligned}$$

Differentiate  $F(s)$  once

# Multiplication by $t^n$



## CONTINUE....

b)  $L\{3te^{-3t}\}$

**STEP 1:**  $f(t) = 3e^{-3t}$ ,  $n = 1$  (refer to  $t$ )

**STEP 2:** Find  $f(t) = 3e^{-3t}$  by using Table of Laplace Transform

$$L\{3e^{-3t}\} = 3 \left\{ \frac{1}{s+3} \right\}$$

$$\therefore F(s) = \frac{3}{s+3}$$

$$3e^{-3t}$$

$$\text{Refer F5: } \frac{1}{s+a}$$

**STEP 3:** Solve the equation by using **F8**:  $n$  from **STEP 1**,  $F(s)$  from **STEP 2**

$$\text{F8 } (-1)^n \frac{d^n}{ds^n} [F(s)]$$

$$L\{3te^{-3t}\} = (-1)^1 \frac{d^1}{ds^1} \left[ \frac{3}{s+3} \right]$$

Differentiate  $F(s)$  once

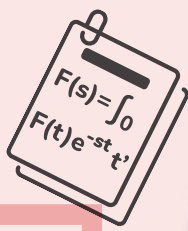
$$= (-1) \frac{d}{ds} [3(s+3)^{-1}]$$

$$= (-1) (-1) 3(s+3)^{-2} (1)$$

$$= 3(s+3)^{-2}$$

$$= \frac{3}{(s+3)^2}$$

# Multiplication by $t^n$



## CONTINUE...

c)  $L\{t^2 e^{5t}\}$

**STEP 1:**  $f(t) = e^{5t}$ ,  $n = 2$  (refer to  $t$ )

**STEP 2:** Find  $f(t) = e^{5t}$  by using Table of Laplace Transform

$$L\{e^{5t}\} = \frac{1}{s-5}$$
$$\therefore F(s) = \frac{1}{s-5}$$

$e^{5t}$  Refer F4:  $\frac{1}{s-a}$

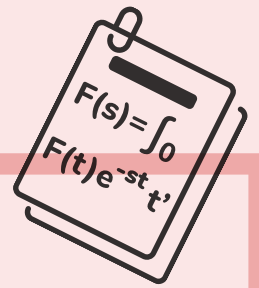
**STEP 3:** Solve the equation by using **F8**:  $n$  from **STEP 1**,  $F(s)$  from **STEP 2**

**F8**  $(-1)^n \frac{d^n}{ds^n} [F(s)]$

$$\begin{aligned} L\{t^2 e^{5t}\} &= (-1)^2 \frac{d^2}{ds^2} \left[ \frac{1}{s-5} \right] \\ &= (-1)^2 \frac{d^2}{ds^2} [(s-5)^{-1}] \\ &= (1) \frac{d}{ds} [(-1)(s-5)^{-2}(1)] \\ &= \frac{d}{ds} [-1(s-5)^{-2}] \\ &= (-2) \times -1(s-5)^{-3}(1) \\ &= 2(s-5)^{-3} \\ &= \frac{2}{(s-5)^3} \end{aligned}$$

Differentiate  $F(s)$  twice

# Multiplication by $t^n$



## EXAMPLE 11

By using the **multiplication by  $t^n$  theorem**, find the following Laplace transform:

- a)  $L\{-7t^2\}$
- b)  $L\{2t \sin 4t\}$

### SOLUTION:

a)  $L\{-7t^2\}$

**STEP 1:**  $f(t) = -7$ ,  $n = 2$  (refer to  $t$ )

**STEP 2:** Find  $f(t) = -7$  by using Table of Laplace Transform

$$L\{-7\} = \frac{-7}{s}$$
$$\therefore F(s) = \frac{-7}{s}$$

-7

Refer F1:  $\frac{a}{s}$

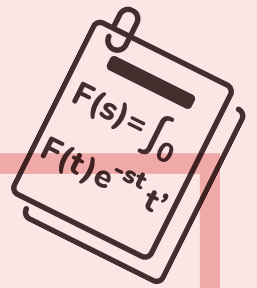
**STEP 3:** Solve the equation by using **F8**:  $n$  from **STEP 1**,  $F(s)$  from **STEP 2**

**F8**  $(-1)^n \frac{d^n}{ds^n} [F(s)]$

$$\begin{aligned} L\{-7t^2\} &= (-1)^2 \frac{d^2}{ds^2} \left[ \frac{-7}{s} \right] \\ &= (-1)^2 \frac{d^2}{ds^2} [-7s^{-1}] \\ &= (1) \frac{d}{ds} [(-1)(-7s^{-2})] \\ &= \frac{d}{ds} [7s^{-2}] \\ &= (-2)(7s^{-3}) \\ &= -14s^{-3} \\ &= \frac{-14}{s^3} \end{aligned}$$

Differentiate  $F(s)$  twice

# Multiplication by $t^n$



**CONTINUE...**

b)  $L\{2t \sin 4t\}$

**STEP 1:**  $f(t) = 2\sin 4t$ ,  $n = 1$  (refer to  $t$ )

**STEP 2:** Find  $f(t) = 2\sin 4t$  by using Table of Laplace Transform

$$L\{2\sin 4t\} = 2 \left\{ \frac{4}{s^2 + 4^2} \right\}$$
$$\therefore F(s) = \frac{8}{s^2 + 16}$$

$2\sin 4t$  Refer F9:  $\frac{\omega}{s^2 + \omega^2}$

**STEP 3:** Solve the equation by using **F8**:  $n$  from **STEP 1**,  $F(s)$  from **STEP 2**

**F8**  $(-1)^n \frac{d^n}{ds^n} [F(s)]$

$$L\{2t \sin 4t\} = (-1)^1 \frac{d^1}{ds^1} \left[ \frac{8}{s^2 + 16} \right]$$

Differentiate  $F(s)$  once

$$= (-1) \frac{d}{ds} [8(s^2 + 16)^{-1}]$$
$$= (-1) (-1) 8(s^2 + 16)^{-2} (2s)$$
$$= 16s(s^2 + 16)^{-2}$$
$$= \frac{16s}{(s^2 + 16)^2}$$

## LET'S PRACTISE!

Find the following Laplace transforms using the **theorem of multiplication by  $t^n$**  :

a.  $L\{\sqrt{2}t\}$

**Answer:**  $\frac{\sqrt{2}}{s^2}$

b.  $L\{te^{-5t}\}$

**Answer:**  $\frac{1}{(s+5)^2}$

c.  $L\{2t \sin 9t\}$

**Answer:**  $\frac{36s}{(s^2+81)^2}$

d.  $L\{t^2e^{7t}\}$

**Answer:**  $\frac{2}{(s-7)^3}$

e.  $L\{t \cos 3t\}$

**Answer:**  $\frac{s^2-9}{(s^2+9)^2}$

f.  $L\{5t^2e^{6t}\}$

**Answer:**  $\frac{10}{(s-6)^3}$

## POP QUIZ

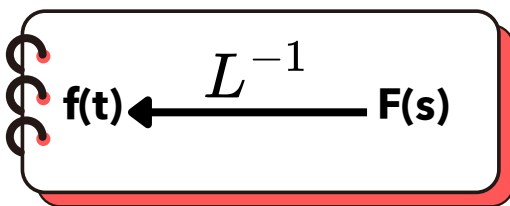
To find out your level of understanding, let's take a pop quiz



POP QUIZ MULTIPLY

# INVERSE LAPLACE TRANSFORM

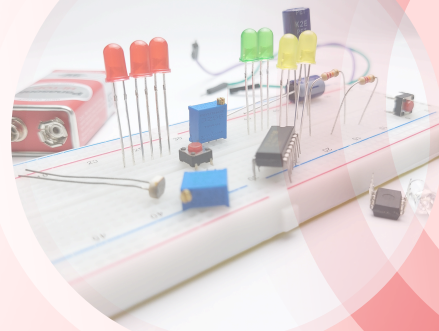
## Understanding the Inverse Laplace Transform



Inverse Laplace transform is the reverse process of a Laplace transform.

If  $L\{f(t)\} = F(s)$ , then  $f(t)$  is called the inverse Laplace transform of  $F(s)$  and is written as  $L^{-1}\{F(s)\} = f(t)$ ,

where  $L^{-1}$  is the operator of the inverse Laplace transform.



### Example 12

Find the Inverse Laplace Transform for  $F(s) = \frac{7}{s}$

**SOLUTION:**

$$F(s) = \frac{7}{s}$$

For example 15, refer table of Laplace no. (1):

$$f(t) = L^{-1}\left\{\frac{7}{s}\right\} = 7$$

No.	$f(t)$	$F(s)$
1.	$a$	$\frac{a}{s}$
2.	$at$	$\frac{a}{s^2}$
3.	$t^n$	$\frac{n!}{s^{n+1}}$
4.	$e^{at}$	$\frac{1}{s-a}$
5.	$e^{-at}$	$\frac{1}{s+a}$

## Example 13

Find the Inverse Laplace Transform for :

$$F(s) = \frac{5}{2s - 10}$$

**SOLUTION:**

For example 16,  
refer table of Laplace  
no. (4):

$$\begin{aligned} f(t) &= L^{-1} \left\{ \frac{5}{2s - 10} \right\} \\ &= L^{-1} \left\{ \frac{5}{2(s - 5)} \right\} \\ &= \frac{5}{2} e^{5t} \end{aligned}$$

No.	$f(t)$	$F(s)$
1.	$a$	$\frac{a}{s}$
2.	$at$	$\frac{a}{s^2}$
3.	$t^n$	$\frac{n!}{s^{n+1}}$
4.	$e^{at}$	$\frac{1}{s - a}$
5.	$e^{-at}$	$\frac{1}{s + a}$

## Example 14

Find the inverse Laplace transform for:

a.  $F(s) = \frac{6}{s^2 + 9}$

b.  $F(s) = \frac{4s}{s^2 - 49}$

**SOLUTION:**

a.  $F(s) = \frac{6}{s^2 + 9}$

b.  $F(s) = \frac{4s}{s^2 - 49}$

For question a,  
refer table of Laplace  
no. (9):

9.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
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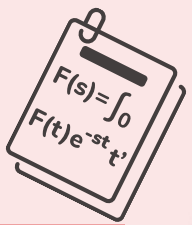
$$\begin{aligned} f(t) &= L^{-1} \left\{ \frac{6}{s^2 + 9} \right\} \\ &= L^{-1} \left\{ \frac{2 \times 3}{s^2 + 3^2} \right\} \\ &= 2L^{-1} \left\{ \frac{3}{s^2 + 3^2} \right\} \\ &= 2 \sin 3t \end{aligned}$$

For question b,  
refer table of Laplace  
no. (16):

16.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
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$$\begin{aligned} f(t) &= L^{-1} \left\{ \frac{4s}{s^2 - 49} \right\} \\ &= 4L^{-1} \left\{ \frac{s}{s^2 - 7^2} \right\} \\ &= 4 \cosh 7t \end{aligned}$$

## LET'S PRACTISE!



Find the inverse Laplace transform for the following expressions:

a.  $F(s) = \frac{5s}{s^2 + 2}$

**Answer:**  $f(t) = 5\cos\sqrt{2}t$

b.  $F(s) = \frac{1}{s^5}$

**Answer:**  $f(t) = \frac{t^4}{24}$

c.  $F(s) = \frac{15}{3s - 4}$

**Answer:**  $f(t) = 5e^{\frac{4}{3}t}$

d.  $F(s) = \frac{s}{s^2 + 121}$

**Answer:**  $f(t) = \cos 11t$

e.  $F(s) = \frac{4s}{9s^2 - 25}$

**Answer:**  $f(t) = \frac{4}{9}\cosh \frac{5}{3}t$

f.  $F(s) = \frac{2s}{16s^2 + 9}$

**Answer:**  $f(t) = \frac{1}{8}\cos \frac{3}{4}t$

## POP QUIZ

To find out your level of understanding, let's take a pop quiz



POP QUIZ  
INVERSE LAPLACE

# Inverse Laplace transform using Theorem 1: Linearity of Laplace Transform

The property of linearity of Laplace transforms can be used to find the inverse of a Laplace transform since the inverse Laplace transform is a linear operation.

## Example 15

Find the inverse Laplace transform using the property of linearity of Laplace transform.

a.  $L^{-1}\left\{\frac{s-3}{4(s^2+9)}\right\}$

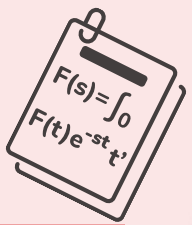
b.  $L^{-1}\left\{\frac{2s+7}{s^2-49}\right\}$

**SOLUTION:**

a. 
$$\begin{aligned} L^{-1}\left\{\frac{s-3}{4(s^2+9)}\right\} &= \left[\frac{1}{4}\left(\frac{s}{s^2+3^2}\right)\right] - \left[\frac{1}{4}\left(\frac{3}{s^2+3^2}\right)\right] \\ &= \frac{1}{4}\cos 3t - \frac{1}{4}\sin 3t \end{aligned}$$

b. 
$$\begin{aligned} L^{-1}\left\{\frac{2s+7}{s^2-49}\right\} &= 2L^{-1}\left\{\frac{s}{(s^2-7^2)}\right\} + L^{-1}\left\{\frac{7}{s^2-7^2}\right\} \\ &= 2\cosh 7t + \sinh 7t \end{aligned}$$

## LET'S PRACTISE!



Find the inverse Laplace transform for the following expressions using the property of linearity of Laplace transforms:

a.  $L^{-1}\left\{\frac{3s+8}{s^2+4}\right\}$

**Answer:**  $3 \cos 2t + 4 \sin 2t$

b.  $L^{-1}\left\{\frac{4s+7}{s^2+9}\right\}$

**Answer:**  $4 \cos 3t + \frac{7}{3} \sin 3t$

c.  $L^{-1}\left\{\frac{2s+5}{s^2-25}\right\}$

**Answer:**  $2 \cosh 5t + \sinh 5t$

d.  $L^{-1}\left\{\frac{6s+4}{s^2+16}\right\}$

**Answer:**  $6 \cos 4t + \sin 4t$

e.  $L^{-1}\left\{\frac{2s+\sqrt{3}}{3s^2+4}\right\}$

**Answer:**  $\frac{2}{3} \cos \frac{2}{\sqrt{3}}t + \frac{1}{2} \sin \frac{2}{\sqrt{3}}t$

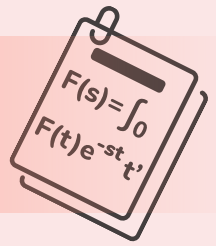
## POP QUIZ

To find out your level of understanding, let's take a pop quiz



POP QUIZ INVERSE  
LAPLACE - LINEARITY

# Inverse Laplace Transform using Theorem 2: First Shifting



The first shifting theorem can be used to find the inverse of a Laplace transform

## Example 16

Find  $L^{-1}\left\{\frac{5}{2s-6}\right\}$  using the first shifting theorem

**SOLUTION:**

$$\begin{aligned} L^{-1}\left\{\frac{5}{2s-6}\right\} &= L^{-1}\left\{\frac{5}{2(s-3)}\right\} \\ &= \frac{5}{2} L^{-1}\left\{\frac{1}{(s-3)}\right\} \\ &= \frac{5}{2} e^{3t} \left\{\frac{1}{s}\right\} \\ &= \frac{5}{2} e^{3t} \end{aligned}$$

Match F(S) with the right formula, refer table of Laplace no. (4):

4.	$e^{at}$	$\frac{1}{s-a}$
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## Example 17

Find  $L^{-1}\left\{\frac{7}{(s+5)^2-4}\right\}$  using the first shifting theorem

**SOLUTION:**

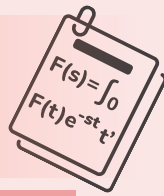
$$L^{-1}\left\{\frac{7}{(s+5)^2-4}\right\} = 7e^{-5t} L^{-1}\left\{\frac{1}{s^2-2^2}\right\}$$

shifting  $\frac{1}{(s+5)}$  into  $e^{-5t}$

Match F(S) with the right formula, refer table of Laplace no. (15):

15.	$\sinh \omega t$	$\frac{\omega}{s^2-\omega^2}$
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$$\begin{aligned} &= \frac{7}{2} e^{-5t} L^{-1}\left\{\frac{2}{s^2-2^2}\right\} \\ &= \frac{7}{2} e^{-5t} \sinh 2t \end{aligned}$$



# Inverse Laplace Transform using Theorem 2: First Shifting

## Example 18

Find  $L^{-1}\left\{\frac{2s}{(s-1)^3}\right\}$  using the first shifting theorem.

**SOLUTION:**

$$\begin{aligned} L^{-1}\left\{\frac{2s}{(s-1)^3}\right\} &= 2L^{-1}\left\{\frac{(s-1)+1}{(s-1)^3}\right\} = 2L^{-1}\left\{\frac{(s-1)}{(s-1)^3} + \frac{1}{(s-1)^3}\right\} \\ &= 2L^{-1}\left\{\frac{1}{(s-1)^2} + \frac{1}{(s-1)^3}\right\} = 2e^t L^{-1}\left\{\frac{1}{s^2} + \frac{1}{s^3}\right\} \\ &= 2e^t \left[ L^{-1}\left\{\frac{1}{s^2}\right\} + L^{-1}\left\{\frac{1}{s^3}\right\} \right] \\ &= 2e^t \left[ t + \frac{t^2}{2!} \right] = 2e^t \left[ t + \frac{t^2}{2} \right] \\ &= 2te^t + t^2 e^t \end{aligned}$$

Match F(S) with the right formula, refer table of Laplace no. 3 & 4:

3.	$t^n$	$\frac{n!}{s^{n+1}}$
4.	$e^{at}$	$\frac{1}{s-a}$

## Example 19

Find  $L^{-1}\left\{\frac{s+3}{s^2-8s+20}\right\}$  using the first shifting theorem.

**SOLUTION:**

$$s^2 - 8s + 20; \quad a = 1, b = -8, c = 20 \text{ (coefficient of } s^2 \text{ must be 1)}$$

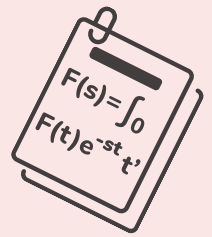
Using completing the square method,  
 $(s - \frac{8}{2})^2 - (\frac{-8}{2})^2 + 20 = (s - 4)^2 + 4$

$$\begin{aligned} L^{-1}\left\{\frac{s+3}{(s-4)^2+4}\right\} &= L^{-1}\left\{\frac{s}{(s-4)^2+2^2}\right\} + L^{-1}\left\{\frac{3}{(s-4)^2+2^2}\right\} \\ &= L^{-1}\left\{\frac{(s-4)+4}{(s-4)^2+2^2}\right\} + 3L^{-1}\left\{\frac{1}{(s-4)^2+2^2} \times \frac{2}{2}\right\} \\ &= L^{-1}\left\{\frac{(s-4)}{(s-4)^2+2^2}\right\} + L^{-1}\left\{\frac{4}{(s-4)^2+2^2}\right\} + 3L^{-1}\left\{\frac{1}{(s-4)^2+2^2} \times \frac{2}{2}\right\} \\ &= e^{4t} L^{-1}\left\{\frac{s}{s^2+2^2}\right\} + 2e^{4t} L^{-1}\left\{\frac{2}{s^2+2^2}\right\} + \frac{3}{2} e^{4t} L^{-1}\left\{\frac{2}{s^2+2^2}\right\} \\ &= e^{4t} \cos 2t + 2e^{4t} \sin 2t + \frac{3}{2} e^{4t} \sin 2t \\ &= e^{4t} \cos 2t + \frac{7}{2} e^{4t} \sin 2t \end{aligned}$$

Match F(S) with the right formula, refer table of Laplace no. 9 & 10:

9.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
10.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$

## LET'S PRACTISE!



Find the inverse Laplace transform for the following expressions using the first shifting theorem

a.  $L^{-1}\left\{\frac{6}{(s-1)^2+4}\right\}$

**Answer:**  $3e^t \sin 2t$

b.  $L^{-1}\left\{\frac{3}{(s+2)^3}\right\}$

**Answer:**  $\frac{3}{2}e^{-2t}t^2$

c.  $L^{-1}\left\{\frac{3s}{(s-1)^4}\right\}$

**Answer:**  $\frac{3}{2}t^2e^t + \frac{1}{2}t^3e^t$

d.  $L^{-1}\left\{\frac{4s}{(s+3)^2}\right\}$

**Answer:**  $4e^{-3t} - 12te^{-3t}$

e.  $L^{-1}\left\{\frac{2s+3}{s^2-4s+20}\right\}$

**Answer:**  $2e^{2t} \cos 4t + \frac{7}{4}e^{2t} \sin 4t$

## POP QUIZ

To find out your level of understanding, let's take a pop quiz



POP QUIZ INVERSE  
LAPLACE - FIRST SHIFTING

## References


- **John B. (2014). Higher Engineering mathematics (7th ed.) . Routledge, New York. (ISBN 978-415-66282-6)**
- **Derek, B. (2020). Probability and statistics: Concepts and applications . Willford Press. (ISBN 978-1682857908)**
- **Sudhir , K., P. (2020). Engineering mathematics with applications. CBS Publishers & Distributors Pvt Limited, India. (ISBN 978-9389688870)**
- **Stroud, K., A. & Dexter, J., B. (2020). Engineering mathematics (8th ed.) . Red Globe Press.**

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Mathematics is not about  
numbers, it's about thinking  
keep exploring!